**Question 1.** Evaluate the cube root of [ z=27cis(240^ \circ)  ](https://my.uopeople.edu/filter/tex/displaytex.php?texexp=%20z%3D27cis(240%5e%20\circ)%20%20). Then raise them to the cube. Show the steps of your reasoning.

**Solution:**  
  
**Step 1: De Moivre's Theorem for Cube Roots**

De Moivre's Theorem states that for any complex number *r* cis(*θ*) and any positive integer *n*:

*(r* cis(*θ*) )n= *rn* cis(*nθ*)

For the cube root (*n* = 3), the formula becomes:

(*r* cis(*θ*) )1/3 =

where *k* is an integer. In this case, we have *r* = 27 and *θ* = 240.

**Step 2: Find the Cube Root of �*z***

The cube root of ***z* is 3 cis(80).**

**Step 3: Raise the Result to the Cube**

Now, we raise this result to the cube:

(3 cis(80) )3

Using De Moivre's Theorem:

33 cis(3.80)

**27 cis(240)**

The cube root of ***z* = 27 cis(240) is 3 cis(80),** and raising this to the cube gives

**27 cis(240),** which is the same as the original *z*.

**Question 2**. Evaluate [  [ \sqrt[5]{3} ( \frac{ \sqrt{3}}{2}+ \frac{i}{2} ) ]^{10}  ](https://my.uopeople.edu/filter/tex/displaytex.php?texexp=%20%20%5b%20\sqrt%5b5%5d%7b3%7d%20(%20\frac%7b%20\sqrt%7b3%7d%7d%7b2%7d%2B%20\frac%7bi%7d%7b2%7d%20)%20%5d%5e%7b10%7d%20%20).

**Solution:**

[]10

**Step 1: Express in Polar Form**

First, express the complex number within the brackets in polar form. For a complex number *a* + *b*, the polar form is *r* cis(*θ*), where *r* = and .

**In this case:**

*a* = and *b* =

**Calculate *r*:**

**r =**

**Calculate :**

**=**

So, the expression in polar form is **cis(​).**

**Step 2: Apply De Moivre's Theorem**

Now, apply De Moivre's Theorem for the power of 10:

[1 cis(​)]10

110 cis(10.)

**cis()**

so the result of []10 in polar form is **cis()**

**Question 3**: Find  [  \frac{z_1}{z_2}  ](https://my.uopeople.edu/filter/tex/displaytex.php?texexp=%20%20\frac%7bz_1%7d%7bz_2%7d%20%20)  in polar form:

[ z_1=21cis(135^ \circ)  ](https://my.uopeople.edu/filter/tex/displaytex.php?texexp=%20z_1%3D21cis(135%5e%20\circ)%20%20)          [ z_2=3cis(75^ \circ) ](https://my.uopeople.edu/filter/tex/displaytex.php?texexp=%20z_2%3D3cis(75%5e%20\circ)%20)

**Solution:**  
**Given information:**z1 = 21 cis(135)

z2 = 3 cis(75)

**Step 1: Write ​​ in Rectangular Form**

To find the quotient of two complex numbers in polar form, we'll first convert them to rectangular form using Euler's formula.

**For *z*1​:**

*z*1 ​= 21 cis(135)

Using Euler's formula: cos(*θ*)+i sin(*θ*), we can write *z*1​ in rectangular form:

*z*1 ​= 21(cos(135) + *i* sin(135))

**Similarly, for *z*2​:**

*z*2​ = 3 cis(75)

*z*2 ​= 3 (cos(75) + isin(75))

Now, we write **​​** ​​ in rectangular form:

​​ =

**Step 2: Simplify the Expression**

To simplify, multiply the numerator and denominator by the conjugate of the denominator to get rid of the complex denominator.

​​ = x

Simplify further by using trigonometric identities.

**Step 3: Simplify Trigonometric Expressions**

​​ =

Simplify each term by applying trigonometric identities.

For example, cos(-75) = cos(75) and sin(-75) = -sin(75)

**Step 4: Perform Multiplications**

​​ =

**Step 5: Combine Real and Imaginary Parts**

Combine the real and imaginary parts to get the final expression.

​​ =

​​ =

​​ = +

**Step 6: Convert Back to Polar Form**

Convert the result back to polar form using the polar coordinate formula:

r =

Where *x* is the real part and *y* is the imaginary part.

For the real part: *x* =

For the imaginary part: *y* =   
**calculate r:**

r =

r =

**r = 12.124357**

**Calculate**

The polar form of ​​ is: